

Computer Science 360
Final Examination
(Open Text Books and Notes)

Time: 3 hours
Marks (Total 100)

December 9, 2003

- 8** 1. Let L be a regular language, then any subset of L is also a regular language. If the above statement is true prove it, if not give a counterexample.
- 10** 2. For each of the following statements indicate whether it is true or false. Give a reason for each decision.
- a. The halting problem can only be solved using nondeterministic Turing machines.
 - b. All problems in NP can be solved on a Turing machine.
 - c. No polynomial time deterministic algorithms are known for NP-complete problems.
 - d. There exists a problem in P that is not contained in NP .
 - e. A deterministic Turing machine can solve all problems that a nondeterministic pushdown automaton can solve.
- 12** 3. Construct a pushdown automaton for the language
$$L = \{a^m b^n \mid n < m \text{ or } n > 2m\}$$
- 16** 4. If L_1 and L_2 are regular languages over the alphabet Σ , prove that the language
$$L_3 = \{xz \mid w, x, y, z \in \Sigma^* \text{ and } xy \in L_1 \text{ and } wz \in L_2\}$$
 is accepted by a finite automaton.
- 16** 5. Consider a divide-and-conquer algorithm for computing the polygonal boundary of the union of a set S of n triangles containing the origin. If the divide step divides S arbitrarily into two equal sized subsets:
- a. What would the conquer step be?
 - b. What would the merge step be?
 - c. Give a recurrence for the running time of the overall algorithm.
- 18** 6. Given an undirected graph G in which each edge is colored either blue or red, write an efficient algorithm to determine whether or not each pair of vertices in G is connected by a path that contains any number of blue edges but at most one red edge.

- 20** 7. [Minimum Set Cover] Given a collection C of subsets of a finite set S , the minimum set cover problem is to find a minimum cardinality subset $C' \subseteq C$ such that every element of S belongs to at least one member of C' .
- Show that a greedy algorithm that first chooses the sets in C of greatest cardinality does not always find the minimum set cover.
 - Give a recursive backtracking algorithm for the minimum set cover problem.